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# A New Kind of Three-Dimensional Anamorphosis

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## Abstract

We define a new kind of anamorphosis, where the distorted image is a three-dimensional object, whose reflection in a mirror restores the hidden design. A chain of softwares and programming languages are used in order to build the sculpture, using a 3D printer. This work, through the genesis of the idea, the elaboration of software tools and solutions, and the interaction between the two partners involved, illustrates how fruitful a collaboration between artists and scientists can be.

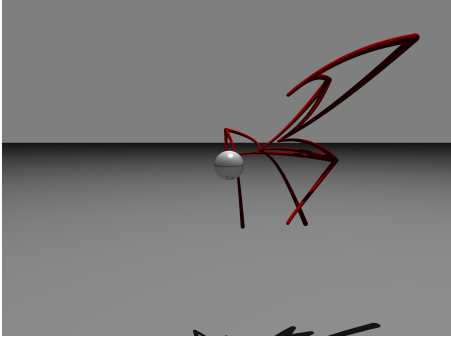
## 1 Introduction

Catoptric anamorphoses are distorted images needing to be seen through a mirror, and from a specific point of view, in order to reconstruct the original image. Anamorphoses appeared in Europe at the time of Renaissance, when artists and scientists discovered the laws of perspective. A complete history of anamorphosis can be found in Jurgis Baltru aitis' *Anamorphoses ou Thaumaturgus opticus* [1].

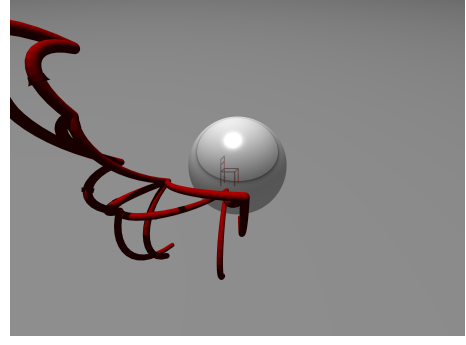
The setting-up of an anamorphosis needs three items : a mirror, an observer, and the locus where the distorted image lays, which I will name *surface of distortion*. In a paper presented at Bridges 2010 [3], I described a procedure for testing and constructing catoptric anamorphoses in the general case. This method can compute the distorted image corresponding to an anamorphosis setting and print it, in order to obtain a real size version of the anamorphosis. In [3], the surface of distortion was either a flat or a developable surface. Extensions of the method can also help define anamorphoses where the surface of distortion is no longer developable: we then lose the possibility of printing the distorted design, but are still able to achieve it by directly drawing on the surface, using the information returned by the program.

In October 2010, James Hopkins, a British sculptor [5], asked me whether it would be possible to compute the shape of a three-dimensional wired form sculpture which would represent a chair when seen through a spherical mirror.

This challenging question could have been asked in another way : "Is it possible to replace the *surface of distortion* with something like a *volume of distortion* ?". The answer is "yes", and figures 1 and 2 prove it can be done. The rest of the paper will explain how we solved this problem. Section 2 will survey previous similar works. Section 3 recalls the method described in [3]. Section 4 explains how this general method was modified in order to generate three-dimensional distorted objects. Section 5 shows the application of the method on an example. Section 6 lists a non-exhaustive set of possible directions to explore, and conclude this paper.



**Figure 1 :** *Reflected chair : the 3D sculpture.*



**Figure 2 :** *The chair seen into the sphere.*

## 2 Previous Works

Among catoptric anamorphoses, those using cylindrical or conical mirrors are current and well studied ([6], [7]). Little has been done for other mirrors shapes, and even less concerning three-dimensional distorted sculptures (instead of distorted flat images): In 1984, Fujio Watanabe [8] exhibited two artworks, where a central conical mirror was surrounded by a plastic carved cylinder. The image of this cylinder, when viewed through the mirror, revealed a human face and a skull. Stella Battaglia and Gianni Miglietta [2] use spherical mirrors surrounded with three dimensional sculptures. James Hopkins has recently made a sculpture where a distorted three-dimensional glass is reflected correctly in a cylindrical reflective bottle. Those three examples are the only ones known by me so far.

## 3 A General Method for the Construction of Catoptric Anamorphoses

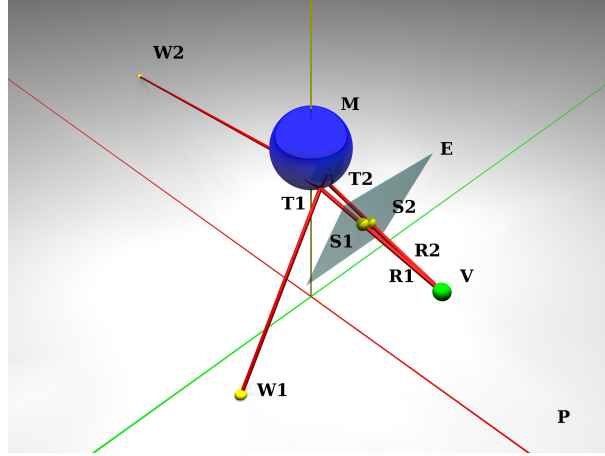
Let us recall the general principle which gave rise to the method presented in [3] :

Figure 3 illustrates the principle behind the method. The conceper of the anamorphosis installation wants the observer, placed at position  $V$ , to see a correct image in a mirror  $M$ . This image is obtained by reflection of a distorted image laying on a surface  $P$ . The image can be thought as laying on a virtual screen  $E$ . Let  $S_1$  be a pixel from the image on  $E$ : when the observer looks at  $S_1$ , ray  $R_1$ , passing through  $V$  and  $S_1$ , hits the mirror  $M$  at point  $T_1$ . This ray is reflected and hits the plane at point  $W_1$ . Reversing the direction of  $R_1$ , we can say that  $W_1$  is seen by the observer, who looks in the direction of the mirror, in the exact direction of  $S_1$ . This reasoning is valid for each pixel. Then the distorted image is the union of all the  $W_i$ , each of them associated with a pixel  $S_i$  of the original image. Screen  $E$ , which was useful for the demonstration, can be removed from the installation. Pixel  $S_2$  shows how the image can be distorted : two neighbouring pixels in the original image can result in very distant points on the surface of distortion.

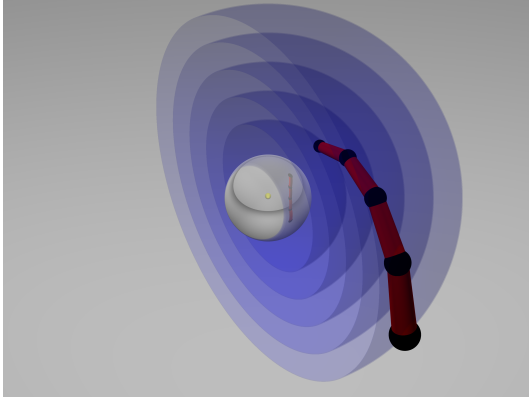
In fewer words, computing a distorted image for an anamorphosis setting consists in following the light rays outgoing from the observer's eye, through the desired image, hitting the mirror and then the surface of distortion. Each such ray defines a pixel on the surface of distortion.

## 4 Adapting the General Method

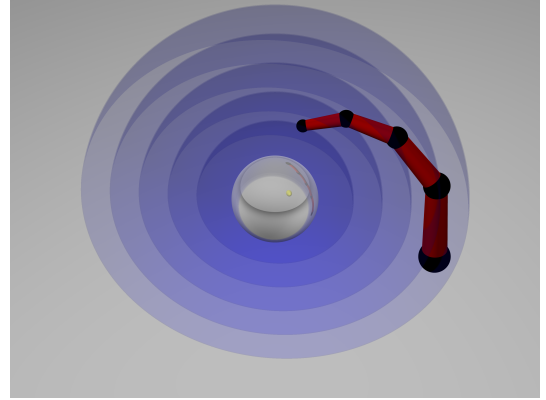
Our problem is slightly different. If we want the distorted object to be a three-dimensional object, we must replace the notion of *surface of distortion* with something else. We decided to define a set of  $N$  surfaces of distortion  $\{S_0, \dots, S_{N-1}\}$ . The image to be distorted is divided into line segments (a 'wired' design). Each



**Figure 3:** *The Catoptric Anamorphosis Principle*



**Figure 4:** *3D anamorphosis method : the line segment in the mirror is the reflection of the 3D curve. The yellow sphere in the center of the mirror is the observer's eye.*



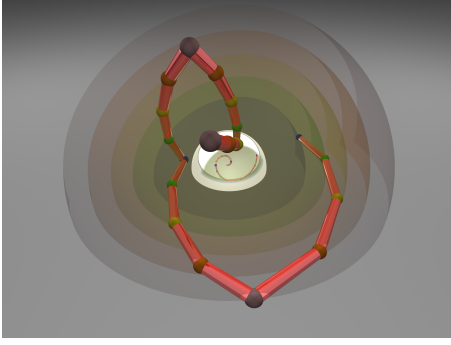
**Figure 5:** *The same scene seen from another point of view.*

segment is itself divided into  $N - 1$  sub-segments, leading to a set of  $N$  points  $\{P_0, \dots, P_{N-1}\}$  corresponding to the extremities of those  $N - 1$  sub-segments. Using the method described in [3], we compute the image  $P'_i$  of point  $P_i$  on the surface of distortion  $S_i$ . When seen into the mirror, the curve connecting the  $P'_i$  in order is an approximation of the original segment. The approximation becomes better as  $N$  increases.

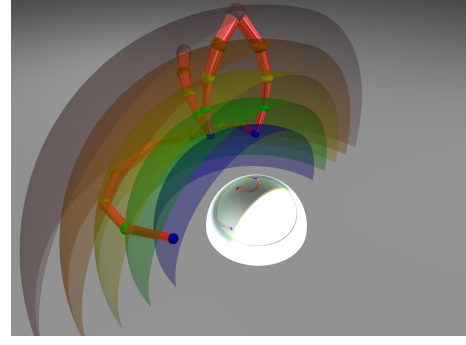
Figures 4 and 5 give an illustration of the process. Five concentric (virtual) spheres are wrapped around the spherical mirror: all together they define the *set of surfaces of distortion*. A vertical line segment is obtained by the reflection of a curve intersecting the five surfaces. Reflections of the black spheres are aligned. For a better result, one must use more than five mirrors (50 mirrors are common). Another point one can notice: the farther the cylinders are from the mirror, the thicker they must be, in order to be perceived with the same thickness when seen by reflection. Hence, the curve connecting the  $P'_i$  is not the union of cylinders, but more precisely a set of (ordered) truncated cones. This poses no problem for the programs used to generate the anamorphosis.

A similar procedure is designed to define the distorted image of an arc of circle, dividing it into  $N - 1$  sub-arcs, and replacing each sub-arc with a truncated cone. Figures 6 and 7 illustrate this process. Hence, combining those two procedures, we can draw designs composed of line segments and arcs of circle, and





**Figure 6:** *Anamorphosis of the approximation of a logarithmic spiral, made with five arcs of circle.*



**Figure 7:** *The 3D sculpture seen from the side. Spheres indicate the intersection of the structure with the volume of distortion.*

then compute their distorted three-dimensional image, given the position of the observer, the shape of the mirror and the set of surfaces of distortion.

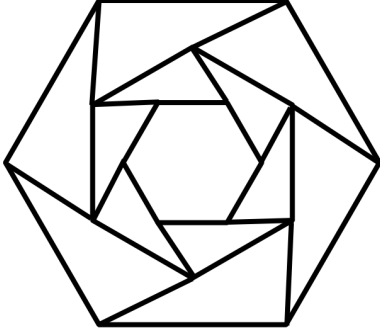
As with the original method, all computations are easy, using the built-in capabilities of a ray-tracing software, whose job is mainly to project and follow rays of light. We use PovRay, a free and open source ray-tracer. Then, outputting the information defining every elementary truncated cone (position, orientation, dimension), we were able to produce a Python script we feed into Blender, a free and open source 3D modeler. We can then scale and manipulate the three-dimensional sculpture, before outputting yet another version of it that we can send to a 3D printer (the only non-free part of the process, if we exclude the cost of computations, but that is the price to pay when coming from virtual to real !).

## 5 A Complete Example

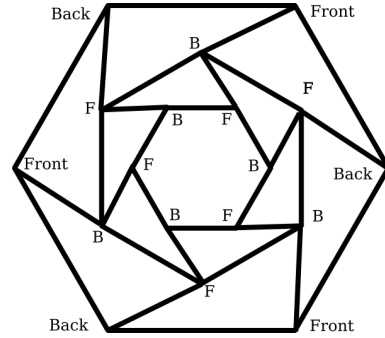
Let us illustrate the process, from the original design to the achieved sculpture. The design we choose is featured in figure 8. Note that the center of this image is free from any line segment : as a practical rule, one must avoid line segments (or arcs of circle) passing through or near the center of the image: this would result in parts of the sculpture located between the observer and the mirror, thus preventing the observer for seeing the reflection. The image is first scaled in order to occupy most of the visible surface of the mirror. We then split the image into line segments, which will correspond to continuous but distorted curves in the final sculpture. For an aesthetic result, we must ensure that a segment extremity will always be either on the first or the last surface of distortion ( $S_0$  or  $S_{N-1}$ ), and that for each line segment, both of its extremities will lay on opposite surfaces of distortion (one near the mirror:  $S_0$ , the other far from it:  $S_{N-1}$ ). This can be done for the chosen design, as shown in figure 9. This is not always the case : just consider an image containing a pentagon or any other odd-sided polygon<sup>1</sup>.

The mirror we choose is a simple spherical one, but the actual mirror's shape does not influence the process in any way. Next, we have to choose the set of surfaces of distortion: here we simply use a set of fifty concentric spheres, with the same center as the mirror. Again, any other choice would have been valid, provided each  $P_i$  has an image on  $S_i$ . Having set the observer's position, we can now call PovRay and produce the virtual anamorphosis setting of figure 10. Figure 11 shows the same installation from another point of view. After having sent the description of the sculpture to a 3D printer, we obtain the object of figure 12. Finally, we validate the whole process by achieving the final installation featured in figure 13. The

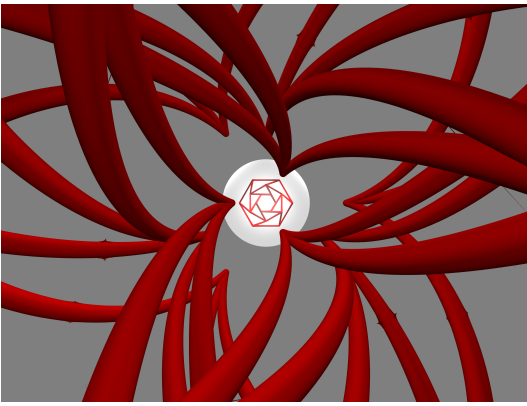
<sup>1</sup>One would object that figure 8 contains triangles, but those are in fact degenerated quadrilaterals, with three points aligned on their hypotenuse.



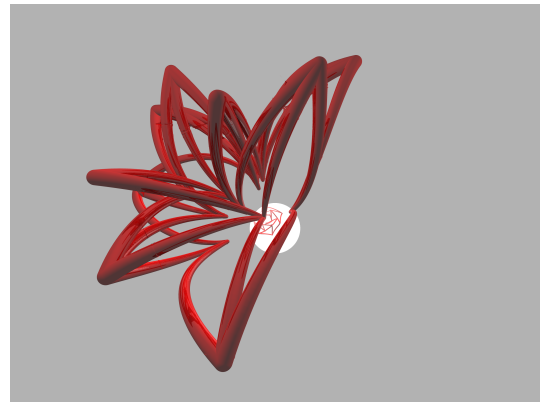
**Figure 8 :** *The original image.*



**Figure 9 :** *Each segment has a Back and a Forward extremity, and the whole labelling is consistent.*



**Figure 10 :** *The virtual installation.*



**Figure 11 :** *A side view of the sculpture.*

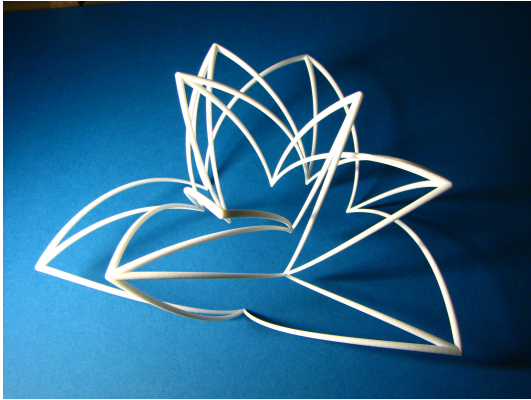
differences between figure 10 and figure 13 are only due to the different focal length of the cameras.

## 6 Conclusion and Perspectives

**A new field of investigation.** After the method has been validated by the concrete achievement of an installation, a wide range of experiments can be conducted. The mirror's shape, the position of the observer and the definition of the set of surfaces of distortion define a kind of cartesian product of possible anamorphosis settings. Each of them can be defined and tested at no cost in less than an hour. It takes no more time to translate the virtual design into a suitable file, and then send it to a 3D printer. Using online 3D printing websites, one then has to wait one week to see the real-world result, with no surprise: everything is like in the virtual version.

**Extending the method even more.** Several variations come immediately to mind: can we replace the wired structure with a solid one, that is to say using surfaces instead of wired skeletons ? What if we use a different (random ?) set of surfaces of distortion for each line segment of the desired image ?

**Colombus' egg.** The process described here uses only one idea, namely following rays of light in the same spirit as the method described by Denis Diderot three hundred years ago in his Encyclopædia[4], cited in [3]. All the rest is only a matter of using existing free computer tools. Like a craftsman able to make new objects because he knows how to manipulate his tools, computer scientists are now able to answer questions quickly, with the help of already written tools. The mass of such programming tools increasingly opens the way to



**Figure 12 :** *The 3D printed sculpture.*



**Figure 13 :** *The final anamorphosis setting.*

experimental mathematics. Questions like "What would this mathematical object look like?", "What if I tune this parameter ?" can now have a quick answer, in the form of a drawing, or even of a three-dimensional object.

**A fruitful collaboration between artist and scientist.** Without the original demand from James Hopkins, I think I would never have thought of this modification of the original method described in [3]. The example presented here has no artistic value in itself : true artists, used to manipulating shapes and materials, might surely extract much more beauty from this arid method.

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